Simulation and Semi-Analytical Approach on Sloshing Mitigation

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Abstract—Free vibrations of liquid in the annular region of a rigid circular cylindrical container with a rigid baffle on the free surface are considered. The liquid inside the container is considered; ideal and incompressible, and the fluid motion is irrotational. In the above-considered geometry, along with assumptions made, the velocity potential is introduced, which satisfies Laplace's equation inside the liquid domain. The boundary value problem (BVP) is formulated using the linear water wave theory. The analytical solution of BVP is obtained in terms of velocity potential with unknown frequency. The velocity potential is used in free surface conditions, which results in a system of homogeneous algebraic equations. The necessary condition for a non-trivial solution of this homogenous system is used to compute the frequencies. Mode shapes of the container in the presence of a rigid baffle are reported using ANSYS software.

Index Terms—Sloshing, frequency, cylindrical container, damping device.

I. INTRODUCTION

Sloshing is an exciting and crucial physical phenomenon observed in storage reservoirs partially filled with liquids. The sloshing is a crucial issue in many engineering applications, one of which is oil storage in tanks. Oil storage tanks are used with rigid/elastic roofs floating on the surface of the oil. The purpose of using floating roofs in tanks is to reduce the evaporation and impose more pressure so that the oil flow rate increases during the tank discharge. The study of liquid-filled tanks with floating structures of various shapes has been a great concern for a great class of researchers.

Many studies have shown that floating roofed vertical cylindrical oil storage tanks become very vulnerable due to the tanks's sloshing phenomenon during earthquakes. Sloshing in oil tanks during an earthquake causes roof collapse/sinking and devastating fires. India has a history of high-frequency earthquakes (M > 8.0), such as four earthquakes between 1897 and 1950. One of the most damaging disasters to Oil industry in past decades was seen in 2003 due to th Tokachi-Oki earthquake (M = 8.0) in Japan.

The Earthquake caused damages due to sloshing in tanks include fires induced by sloshing and sinking of the floating roof. Baffles as a means of suppression devices have been used for a long time to control damage due to sloshing. The effect of rigid baffles with negligible thickness attached to the tank's

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wall are discussed in [1], [2], [3] & [17], and a semi-analytical approach is used to compute the sloshing frequencies and their dependance on various parameters considered. Researchers have not analyzed the floating baffle's effect on the free surface and its impact on vibration properties in all these works.

The concept of floating suppression devices (as baffles) in containers received popularity after the Tokachi-Oki earthquake damage in (2003) due to its high amplitude sloshing waves generated inside the tanks. The sloshing frequencies were excited due to strong ground motion. Though not sufficient research has been done in this direction. Mainly, the studies done in this direction have been discussing the effect. A few studied on sloshing since Tokachi-Oki (2003) are studied in [12], [21], and [22]. Suppression devices (rigid/elastic) are considered most effective to dampen the impact loads in a tank, as shown in [6], [7], [20], and [13]. Few works have been reported where porous and perforated baffles are considered to discuss sloshing. Some of the studies on porous and perforated baffles are shown in [5], [10], [11], and [15]. Non-linear sloshing and its effects have been discussed in [14], [18] and [19]. Some of the studies investigating the dynamic behavior of containers are reported in [4], [9], [16], and [23]. In the presented paper, sloshing frequencies for the first mode and the first vibration modes are reported.

II. MATHEMATICAL MODELING OF THE PROBLEM

A rigid annular circular cylindrical container with outer radius b and inner radius a, which is filled with liquid up to height h, is considered. A ring-shaped baffle of width α with nominal thickness is placed at the free surface centrally. Cylindrical-coordinate system (r, θ, z) , in which z is measured vertically upwards from the mean surface, is considered. The origin O is regarded at the mean surface, and hence z = 0 is the mean position of the free surface of the liquid inside the container. Here, z = -h represents the bottom of the cylinder. Fig. 1 depicts the schematics diagram.

An incompressible, irrotational and inviscid liquid is allowed to perform the motion inside the annular region of the container. Linear water wave theory is used to formulate the problem. Under these assumptions, fluid flow inside the fluid



Fig. 1. Schematic Diagram of the problem

domain can be described in terms of velocity potential $\overline{\Phi}$ from the following governing equation:

Governing Equation:-

$$\nabla^2 \bar{\Phi}(r,\theta,z,t) = 0, \tag{1}$$

which is Laplace's equation in cylindrical coordinates, where

$$\nabla^{2} = \frac{\partial^{2}}{\partial r^{2}} + \frac{1}{r} \frac{\partial}{\partial r} + \frac{1}{r^{2}} \frac{\partial^{2}}{\partial \theta^{2}} + \frac{\partial^{2}}{\partial z^{2}}.$$
 (2)

To investigate the behaviour of liquid, this partial differential equation must be solved subject to the appropriate boundary conditions. The geometry in the consideration is assumed to be rigid. The Boundary condition at the rigid bottom of the container is given by

$$\frac{\partial \bar{\Phi}}{\partial z} = 0, \quad \text{at} \quad z = -h.$$
 (3)

The Boundary conditions at the rigid side wall of the container are given by

$$\frac{\partial \bar{\Phi}}{\partial r} = 0, \quad \text{at} \quad r = a,$$
 (4)

$$\frac{\partial \Phi}{\partial r} = 0, \quad \text{at} \quad r = b.$$
 (5)

The Free surface condition is divided into three conditions due to the insertion of the rigid baffle at the free surface. Followings are the free surface conditions of the given problem:

$$\frac{\partial^2 \bar{\Phi}}{\partial t^2} + g \frac{\partial \bar{\Phi}}{\partial z} = 0, \quad \text{at} \quad z = 0$$

in $a < r < \frac{a+b-\alpha}{2},$ (6)

$$\frac{\partial^2 \bar{\Phi}}{\partial t^2} + g \frac{\partial \bar{\Phi}}{\partial z} = 0, \quad \text{at} \quad z = 0$$

in
$$\frac{a+b+\alpha}{2} < r < b, \tag{7}$$

$$\frac{\partial \bar{\Phi}}{\partial z} = 0, \quad \text{at} \quad z = 0$$

in
$$\frac{a+b-\alpha}{2} \le r \le \frac{a+b+\alpha}{2}, \quad (8)$$

where g is the gravitational constant. In order to solve the above BVP, consider

$$\bar{\Phi}(r,\theta,z,t) = \phi(r,\theta,z) \exp^{\iota \omega t}.$$
(9)

The analytical solution for (1) subject to the boundary conditions (3) - (5) is obtained using separation of variable method and is given by

$$\bar{\Phi}(r,\theta,z,t) = \sum_{m=0}^{\infty} \sum_{n=1}^{\infty} (A_{mn} \cos(m\theta) + B_{mn} \sin(m\theta)) \\ \frac{C_{mn}(\frac{\epsilon_{mn}}{b}r)}{J'_m(\frac{\epsilon_{mn}}{b}a)} \frac{\cosh(\frac{\epsilon_{mn}}{b}(z+h))}{\cosh(\frac{\epsilon_{mn}}{b}h)} \exp^{\iota \omega t},$$
(10)

where,

$$C_{mn}(\frac{\epsilon_{mn}}{b}r) = J'_m(\frac{\epsilon_{mn}}{b}a)Y_m(\frac{\epsilon_{mn}}{b}r) - J_m(\frac{\epsilon_{mn}}{b}r)Y'_m(\frac{\epsilon_{mn}}{b}a),$$
(11)

and ϵ_{mn} are the roots of

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$$J_m'(rac{\epsilon_{mn}}{b}a)Y_m'(\epsilon_{mn})-J_m'(\epsilon_{mn})Y_m'(rac{\epsilon_{mn}}{b}a)=0.$$

The solution given by (10) is used in (6) - (8), to get a system of homogenous equations which is used to compute the sloshing frequencies.

III. NUMERICAL RESULTS

The sloshing frequencies and the first modes of the liquidfree surface are reported for different parameters with the help of numerical simulation,. We compare our numerical results obtained with analytical data from R. A. Ibrahim [8], results obtained by the finite element method, and the analytical method proposed here using system of homogenous equations. Here, all the measurements are considered in the unit meter(m). In the presented paper, outer radius is considered b = 1m, inner radii are different, namely a = 0.5m, a = 0.33333m, a = 0.25m. Also, different values of the width for ring baffles are considered, namely $\alpha = 0.1m, \alpha = 0.2m, \alpha = 0.3m.$

Firstly, the case of no baffle at the liquid-free surface is considered. The first frequency comparison is presented in Table 1 for annular circular cylindrical container without baffle on the liquid-free surface with b = 1m, a = 0.5m, and liquid filling level h = 1m.

The data of numerical simulations testified the accuracy of proposed method. The modes of first axisymmetric vibrations for b = 1m, a = 0.5m and h = 1m in un-baffled structure are shown in Fig. 2 and the frequencies corresponding to first modes are given in Table 2.

Fig. 3 demonstrates mode of un-baffled container for inner outer radii ratio a/b = 1/4.

In second case, a rigid baffle at the free surface of liquid is considered and simulation is done for a system with configuration considered as b = 1m, a = 0.5m and h = 1m. The baffle width is considered $\alpha = 0.1m$. Fig. 4 shows the first mode of liquid vibrations.

Fig. 5 and Fig. 6, show the curves drawn for frequency ω as a function of liquid height h for two different values of baffle width taken as $\alpha = 0$ and $\alpha = 0.1m$, respectively. Graphs for frequencies vs filling level h for inner-outer radii ratio a/b = 1/2 are plotted. It is observed that frequency increases with increasing filling level of liquid. The effect of baffle width is also shown and it is observed that frequency is higher in case of baffle placed at the free surface.

IV. CONCLUSION

A specially designed annular circular cylindrical container with liquid, filled inside the annular region is considered in the presented paper,. The effect of an annular baffle placed centrally at the free surface of the liquid is analyzed. The sloshing frequencies are evaluated for a fixed baffle width, varying fluid height, and radial values of the annular region. Results demonstrate the dependency of frequencies on the width of the baffle and the filling level. It is observed that sloshing frequency increases in the presence of baffle placed at the free surface, resulting in reduced sloshing. The first Mode Shapes are reported of liquid vibration in case of the baffle and without baffle.

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TABLE I VALIDATION OF PROPOSED METHOD

Method	First Frequency
FEM, ANSYS 108000 finite elements	0.5427756321
Analytical Method used in [3]	0.5427673427
Method Presented	0.5427697633

TABLE II FREQUENCIES OF UN-BAFFLED STRUCTURE

Mode	Frequency
1	0.54277
2	0.54277
3	0.81242
4	0.81242



Fig. 2. First Mode of liquid vibrations without baffle at free surface



Fig. 3. First Mode of un-baffled container for a/b=1/4



Fig. 4. First Mode of liquid vibrations with baffle at free surface



Fig. 5. Sloshing frequency vs filling level h for un-baffled container for inner-outer radii ratio a/b = 1/2

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Fig. 6. Sloshing frequency vs filling level h for baffled container for innerouter radii ratio a/b = 1/2 and $\alpha = 0.1m$

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