Sloshing in Two Immiscible Fluids in the Presence of a Suppression Device

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Abstract—The aim of the presented work is to compute sloshing frequencies. Two immiscible fluids of different densities are considered in a cylindrical tank. A rigid baffle is placed at the free surface of the upper fluid. The mathematical formulation is done by dividing the fluid domain into sub-domains. The mathematical formulation is done for each sub-domain to find the analytical solution in each sub-domain. The linear water wave theory is used to find the velocity potentials in each sub-domain. The effects of the fluid heights on the natural sloshing frequencies are studied. The findings are supported by relevant graphs.

Index Terms—Sloshing, Vibration, frequency, cylindrical container, damping device/baffle.

I. INTRODUCTION

Free surface motion inside any partially filled tanks is crucial in the safety and stability of the tank as well as the vehicle carrying the tank. The liquid motion inside the container is termed as liquid Sloshing. Sloshing is encountered in several engineering applications in aerospace engineering, civil engineering and mechanical engineering. The motion of the the free surface in partially liquid-filled containers have been investigated by a large class of researchers.

The free surface flows inside the tanks in rockets, missiles and aircrafts have been the primary focus of extensive research work for many researchers. Numerical treatment of the translational excitation of the free surface motion is reported in [1] and [2]. The study of the oscillations of fluid filled in the fuel tank of space vehicle is reported in [1] and the effects of oscillations on stability of space vehicle are analyzed. Slosh suppression devices known as baffles play a crucial role on the control of sloshing. It is shown that the insertion of rigid baffles at the free surface increases the frequency shifting the resonance to higher values. Some of the studies of linear sloshing in cylindrical tanks of various geometries have been reported in [3], [7], [8], [10], and [17]. The effects of the rigid baffle, placed on the free surface of the liquid, are examined on sloshing frequencies. It is shown that such arrangements can control the sloshing frequencies substantially. Sloshing in a vertical circular cylinder with a rigid baffle placed horizontally and vertically has been examined and it is found that sloshing is affected by the geometry and location of the baffle. A detailed description of the normal modes in containers of different configurations is found in [10]. The sloshing problem

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in a rectangular container has been carried out in [4] The Bateman-Luke variational principle is used to derive a system of ODEs. The work done in [4] has been extended in [5] to discuss the impact of surge and pitch excitations. Sloshing in a rectangular tank has been studied to discuss the dynamic response at various filled levels of the tank in [18]. A 3-dimensional model based on fluid-structure interaction has been developed in [6] to analyze liquid sloshing. The effects of hydrodynamic forces are also observed in the presence of baffles. A study on modeling of sloshing inside a tank has been reported in [9].

In general, the study of the sloshing problem in singlelayer fluid has been the main focus. The sloshing frequencies for a two layer fluid in a tank have been evaluated by the author. A study on sloshing in a tank filled with two-fluids has been reported in [16]. It is found that most undesired forces occur near to the external excitation frequency. A two layer sloshing problem in a tank has been discussed in [11]. An asymptotic expansion method is used to derive a secondorder ODE. The Sloshing inside a tank filled with layered fluid has been reported in [15] with the help of Hamiltonian mathematical model and the findings are validated by laboratory experiments. Some of the studies on sloshing in a container containing three fluids and in the presence of perforated screens are reported in [12], [13], and [14]. In the presented paper, the sloshing in a two-fluid system in a container with a baffle placed on the upper fluid-free surface has been discussed.

II. PROBLEM FORMULATION AND SOLUTION

Two immiscible, inviscid, and incompressible fluids are considered to have irrotational motion in a circular tank. The tank is partially filled with two fluids. The radius of the tank is considered as R_1 . Cylindrical coordinates (r, θ, z) are introduced. Fig. 1 describes the problem.

The presence of baffle on the free surface of the upper fluid and two-fluid make it difficult to find the solution. The fluid domain is divided into four different sub-domains to setup the boundary value problem in each sub-domain. The velocity potential $\overline{\Phi}(r, \theta, z, t)$ in each sub-domain can be written According to assumptions made, velocity potential in each sub-domain satisfies Laplace's equation

$$\frac{\partial^2 \bar{\Phi}_i}{\partial r^2} + \frac{1}{r} \frac{\partial \bar{\Phi}_i}{\partial r} + \frac{1}{r^2} \frac{\partial^2 \bar{\Phi}_i}{\partial \theta^2} + \frac{\partial^2 \bar{\Phi}_i}{\partial z^2} = 0 \qquad \text{in} \quad \Omega_i.$$
(1)

The boundary conditions at rigid boundaries of the container are given by

$$\left(\frac{\partial\bar{\Phi}_1}{\partial r}\right)_{r=R_1} = 0, \quad \left(\frac{\partial\bar{\Phi}_1}{\partial z}\right)_{z=h_1} = 0, \quad (2)$$

$$\left(\frac{\partial \Phi_3}{\partial r}\right)_{r=R_1} = 0, \quad \left(\frac{\partial \Phi_3}{\partial z}\right)_{z=-h_2} = 0, \qquad (3)$$
$$\left(\frac{\partial \bar{\Phi}_4}{\partial z}\right) = 0. \qquad (4)$$

$$\left(\frac{\partial^2 4}{\partial r}\right)_{z=-h_2} = 0.$$

The free surface condition is as follows:

$$\frac{\partial \Phi_2}{\partial z} + \frac{1}{g} \frac{\partial^2 \Phi_2}{\partial t^2} = 0$$
 at $z = h_1$, (5)



The time harmonic potentials in corresponding domains can be written as follows:

$$\bar{\Phi}_i(r,\theta,z,t) = \bar{\Phi}_i(r,\theta,z)e^{\iota\omega t},$$
(6)

where ω denotes the angular wave frequency. Using (6) into (2) - (4), the corresponding boundary conditions can be written as

$$\left(\frac{\partial \bar{\Phi_1}}{\partial r}\right)_{r=R_1} = 0, \quad \left(\frac{\partial \bar{\Phi_1}}{\partial z}\right)_{z=h_1} = 0, \tag{7}$$

$$\left(\frac{\partial \bar{\Phi}_3}{\partial r}\right)_{r=R_1} = 0, \quad \left(\frac{\partial \bar{\Phi}_3}{\partial z}\right)_{z=-h_2} = 0, \tag{8}$$

$$\left(\frac{\partial \Phi_4}{\partial r}\right)_{z=-h_2} = 0. \tag{9}$$

In the periodicity of $\overline{\Phi}$, consider

$$\bar{\Phi}_i = \sum_{m=0}^{\infty} \bar{\Phi}_{im} \cos m\theta, \qquad (10)$$

where m denotes azimuthal mode. The boundary conditions for $\overline{\Phi}_i$ are found using (7) – (9). In order to nondimensionalize the parameters, following dimensionless quantities are introduced :

$$\xi = \frac{r}{R_1}, \quad \eta = \frac{z}{R_1}, \quad \gamma = \frac{R}{R_1}, \quad \beta_1 = \frac{h_1}{R_1}, \quad \beta_2 = \frac{h_2}{R_1}, \quad \omega^* = \omega \sqrt{\frac{R_1}{g}}.$$
 (11)

To solve BVP in each sub-domain, the separation of variable method is used in (r, θ, z) -coordinates for $\overline{\phi}_i$ and the analytical solutions are given in the following forms

$$\bar{\Phi}_{1} = \sum_{n=1}^{\infty} A_{1mn} \cos\left(\frac{n\pi}{\beta_{1}}\eta\right) \times \frac{\left[I_{m}\left(\frac{n\pi}{\beta_{1}}\xi\right) K'_{m}\left(\frac{n\pi}{\beta_{1}}\right) - I'_{m}\left(\frac{n\pi}{\beta_{1}}\right) K_{m}\left(\frac{n\pi}{\beta_{1}}\xi\right)\right]}{K'_{m}\left(\frac{n\pi}{\beta_{1}}\right)} + A_{1m0}\left(\xi^{m} + \xi^{-m}\right)\delta_{m}^{2} + A_{100}, \quad (12)$$

where k_{mn} are the zeros of

$$J'_{m}(k_{mn})Y'_{m}(k_{mn}\gamma) - J'_{m}(k_{mn}\gamma)Y'_{m}(k_{mn}) = 0, \quad (13)$$

$$\bar{\Phi}_2 = \sum_{n=1}^{\infty} A_{2mn} \cosh(\lambda_{mn}\eta) \times J_m(\lambda_{mn}\xi) + A_{200}, \quad (14)$$
(15)

where λ_{mn} are the zeros of

$$J'_m(\lambda_{mn}\gamma) = 0. \tag{16}$$

$$\bar{\Phi}_{3} = \sum_{n=1}^{\infty} A_{3mn} \cos\left(\frac{n\pi}{\beta_{2}}\eta\right) \times \frac{\left[I_{m}\left(\frac{n\pi}{\beta_{2}}\xi\right)K'_{m}\left(\frac{n\pi}{\beta_{2}}\right) - I'_{m}\left(\frac{n\pi}{\beta_{2}}\right)K_{m}\left(\frac{n\pi}{\beta_{2}}\xi\right)\right]}{K'_{m}\left(\frac{n\pi}{\beta_{2}}\right)} + A_{3m0}\left(\xi^{m} + \xi^{-m}\right)\delta_{m}^{2} + A_{300}.$$
 (17)

$$\bar{\Phi}_4 = \sum_{n=1}^{\infty} A_{4mn} \cos\left(\frac{n\pi}{\beta_2}\eta\right) \times I_m\left(\frac{n\pi}{\beta_2}\xi\right) + A_{4m0}\xi^m \delta_m^2 + A_{400}.$$
 (18)

The coefficients A_{imn} are still unknown. Using continuity of pressure and velocity at the interface along with free surface condition, an infinite system of equations is obtained. This homogeneous system is used to compute the frequency and unknown parameters.



III. RESULTS

numerical calculations are done by using MATHEMAT-ICA. The roots of the determinant of the truncated homogeneous system are drawn. The graphs are shown for non-dimensional frequency. Dimensionless sloshing frequency ω_{mn}^{*2} (m = 2; n = 1, 2) is drawn for various γ values.

Fig. 2 describes the behaviour of the first non-dimensional frequency ω_{21}^{*2} versus the upper fluid-height ratio to tank radius. It is shown that ω_{21}^{*2} increases gradually with increasing β_1 . It is also found that there is a significant increment in the value of ω_{21}^{*2} with a decreasing value of γ .



Fig. 2. ω_{21}^{*2} versus β_1 for different values of γ for $\beta_2 = 0.5$, $\rho = 0.63$

Fig. 3 describes the behaviour of ω_{21}^{*2} versus β_1 , β_1 is considered as the upper fluid height ratio to the container radius. Here, $\beta_2 = 0.5$ is considered and the density ratio ρ is kept fixed at 0.63. The curves represents ω_{21}^{*2} for different values of $\gamma = 0.2, 0.4, 0.6$.

Fig. 4 shows the curves for ω_{21}^{*2} against the lower fluid-depth ratio to the tank radius. It is shown that ω_{21}^{*2} increases gradually as the value of the filled level of upper fluid increases. It is observed that ω_{21}^{*2} increases with a decreasing γ . Fig. 5 shows the curves for ω_{22}^{*2} against the lower fluid-depth ratio to the tank radius. Here, non-monotonic behaviour of ω_{22}^{*2} is observed. It is shown that ω_{22}^{*2} initially increases for small values of β_2 and then decreases for higher values of β_2 .

IV. CONCLUSION

In the presented paper, a semi-analytical approach is utilized to discuss the sloshing in a vertical cylindrical tank. The considered tank is filled with two fluids with different densities. The effects of fluid heights of both fluids are examined on the sloshing frequencies when a rigid baffle is inserted on the free surface of the upper fluid.



Fig. 3. ω_{22}^{*2} versus β_1 for different values of γ for $\beta_2 = 0.5$, $\rho = 0.63$



Fig. 4. ω_{21}^{*2} versus β_2 for different values of γ for $\beta_1 = 0.5$, $\rho = 0.63$

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Fig. 5. ω_{22}^{*2} versus β_2 for different values of γ for $\beta_1 = 0.5$, $\rho = 0.63$

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